Problem bank for "Week 10" material: Relative Extrema; Higher Derivatives, Concavity, and the Second Derivative Test

27 Oct 2020

Group 1

textbfApplication to Business and Economics

The demand equation for telephones at one store is

$$p = D(q) = 200e^{-0.1q},$$

where p is the price (in dollars) and q is the quantity of telephones sold per week. Find the values of q and p that maximize revenue.

Group 2

Let

$$f(x) = \frac{x^2}{\ln(x)}.$$

Find the x-value of all points where the f has any relative extrema. Find the value(s) of any relative extrema.

Group 3

Let

$$f(x) = 3x^{5/3} - 15x^{2/3}.$$

Find the x-value of all points where the f has any relative extrema. Find the value(s) of any relative extrema.

Group 4

Find the open intervals where the following functions are concave upward or concave downward. Find any inflection points.

1. $$f(x) = \frac{3}{x-5}$$

2. $$g(x) = \ln(x^2 + 1)$$

Group 5

Find any critical numbers for the functions f below and then use the second derivative test to decide whether the critical numbers lead to relative maxima or relative minima. If $f''(c) = 0$ or $f''(c)$ does not exist for a critical number c, then the second derivative test gives no information. In this case, use the first derivative test instead.

1. $$f(x) = -x^2 - 10x - 25$$

2. $$f(x) = x^{7/3} + x^{4/3}$$

Extra Problems

1. **Application to Life Sciences**

   The mathematical relationship between the age of a captive female moose and its mass can be described by the function

   $$M(t) = 369(0.93)^t t^{0.36}, \quad t \leq 12$$

   where $M(t)$ is the mass of the moose (in kilograms) and t is the age (in years) of the moose. Find the age at which the mass of a female moose is maximized. What is the maximum mass?

   Source: Journal of Wildlife Management.

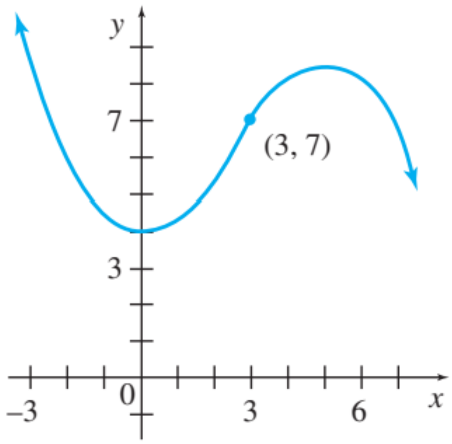
2. **Application to Business and Economics** Let $C(q) = 80 + 18q$ be the cost of producing q units on a product. If the demand equation is $p = 70 - 2q$, find

   (a) the number, q, of units that produces maximum profit

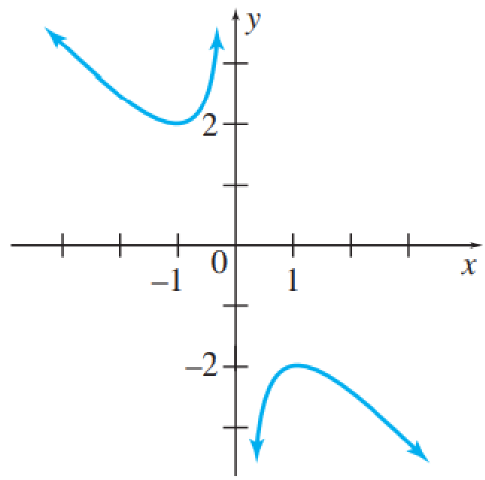
(b) the price, p, per unit that produces maximum profit

(c) the maximum profit, P

3. Find the open intervals where the functions graphed below are concave upward or concave downward. Find any inflection points.



(a)



(b)